

# Modern Semiconductor Devices for Integrated Circuits

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## Part I : Electrons and holes in a semiconductor

### Chapter summary

1. Silicon crystals are formed via covalent bonds. In an intrinsic silicon crystal there are few mobile electrons and holes. Their concentration is equivalent to  $n_i$  ( $\sim 10^{10} \text{ cm}^{-3}$ )
2. Using the bond model one can explain why group V atoms can serve as donors where as group III atoms can serve as acceptors
3. Electrons and holes are the major characters in the play and carry opposite charge. Their mass however is altered from the mass of an electron in vacuum. The altered mass is called effective mass,  $m_n$  and  $m_p$
4. The band model is the tool required for quantitative analysis of semiconductors. From this model one can get the energy gap, E-K diagrams allowing the determination of effective masses, analysis of the energy levels with in the gap and the conduction/valence bands etc
5. Band gap is defined as the separation between the valence and the conduction band. Si 1.1eV, GaAs 1.424eV, InP 1.344eV
6. The valence and the conduction band contain states which contain electrons and holes. This is defined by density of states ( $\#/\text{volume}/\text{energy}$ ) for a certain energy range
7. The distribution of electrons and holes in a semiconductor is given by the fermi-dirac distribution. Fermi-Dirac distribution is dependent on the temperature

8. The fermi level or fermi energy is determined by the available electrons and states in the system.
9. The total number of filled states in the conduction or valence band defined by:  $n = \int D(E) F(E) dE$
10. Thermal agitation gives each energy state a certain probability of being occupied by an electron. That probability is expressed by the reduced fermi function for  $E$  more than a few  $kT$  above the Fermi level,  $E_F$ .
11. The fermi level,  $E_F$ , is related to the density of electrons and holes in the following manner:

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT}$$

12. The effective density of states,  $N_c$  &  $N_v$ , are around  $10^{19} \text{cm}^{-3}$
13. The majority carrier concentrations are (for a N and P type Semiconductor):

$$n = N_d - N_a$$

$$p = N_a - N_d$$

where  $N_d$  &  $N_a$  are the concentration of donors and acceptors which are usually assumed to be completely ionized.

14. The intrinsic carrier concentration,  $n_i$ , is determined as a function of  $E_g$  and  $T$

**Properties of semiconductors** Different surface orientations have different properties such as the rate of oxidation and the electronic quality of the oxide/semiconductor interface. Both the surface orientation and the direction of the current flow along the surface affects the speed performance of a surface based device such as a MOSFET

## 1 Semiconductor

### Silicon

To break a silicon bond an energy of 1.1 eV is required.  $E_{thermal} = 26 \text{ meV}$  at 300 K and 1.1 eV is roughly 90000 K. Therefore doping is done because the energy to relax an electron or hole is  $\sim 50 \text{ meV}$

## GaAs

Each Ga atom has four As atoms and each As atom has four Ga atoms. Group VI elements are donors because they replace As, i.e. S, Se etc and group III elements are acceptors, i.e. Zn. Group IV elements like Si and Ge can be either donors or acceptors depending on their electronegativities. For e.g. silicon is a donor because energetically favorable to replace Ga atoms while Ge is an acceptor because it is energetically favorable to replace As atoms

## 2 Energy Band Model

### Bond Model

Pauli exclusion principle coupled with bringing atoms together is used to derive the band model. A band gap exists where the lower lying band or the lower energy band is called the valence band and the higher lying band or higher energy band is called the conduction band. The gap between is called the band gap. The electrons in a totally filled band does not have a net velocity and does not conduct current and neither does an empty band. Therefore it is only the valence band and the conduction band which conducts current.

The electrons in the valence band are those associated with the covalent bonds in the bond model and the electrons in the conduction band are the conduction or mobile electrons.

Band gaps are determined by measuring the absorption of light. Below the band gap energy the substance is transparent and above the band gap energy the substance absorbs the photon.

Useful relationship to convert photon frequency into energy (eV)  $h\nu = \frac{1.24}{\lambda}$

### Donors and Acceptors in Band model

Donors,  $E_d$ , donate electrons to the conduction band while acceptors,  $E_a$ , accept an electron from the valence band.

### Electrons and Holes

Holes can be thought of as bubbles in liquids, floating up in the energy band and electrons can be thought of as water drops that tend to fall to the lowest energy state in the energy band. This is because in the energy band model the y-axis are states of higher energy for the electrons and lower energy for holes. This is true because the y-axis energy is based on electron charge.

## Effective Mass <https://accounts.google.com/ServiceLogin?service=mail>

In a crystal the electrons and holes interact with a periodic coulombic field. They surf over the periodic potential of the crystal and therefore  $m_n$  and  $m_p$  are not the same as the free electron mass. A complete description of electrons in a crystal must be based on their wave characteristics. The electron wave function will be the solution of a 3-D Schrodinger Equation. The solution will be of the form  $\exp(\pm k \cdot r)$  where  $k$  is the wave vector and is a function of energy. For each  $k$  there is a corresponding energy. Assuming the E-k relationship has a spherical symmetry, an electric field,  $E$ , would accelerate an electron wave packet with:  $a = \frac{qE}{\hbar^2} \frac{d^2 E}{dk^2}$ ; here the concept of effective mass is introduced,  $m_{eff} = \frac{\hbar^2}{d^2 E / dk^2}$ , in order to interpret the acceleration equation as  $F = ma$ .

Effective masses for electrons and holes in a semiconductor:

	Si	GaAs
$\frac{m_n}{m_0}$	0.26	0.068
$\frac{m_p}{m_0}$	0.39	0.50

Each semiconductor has a unique E-k relationship for its conduction band and its valence band both due the unique  $V(r)$ .  $\frac{d^2 E}{dk^2}$  is the curvature of the band. Therefore the effective mass is related to inverse of curvature of the band.

### 3 Density of States

Density of states is used to think of energy bands as a collection of discrete energy states. Here the energy states is defined in quantum mechanical terms as representing spin up and spin down states and a unique solution to the Schrodinger wave equation for the periodic electric potential function of the semiconductor. Each state can hold either one electron or none. Conduction band and valence band density of states is:

$$D_c(E) = \frac{\# \text{ of states in } \Delta E}{\Delta E \times \text{volume}} = \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3} \quad E \geq E_c$$

$$D_v(E) = \frac{\# \text{ of states in } \Delta E}{\Delta E \times \text{volume}} = \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3} \quad E \leq E_v$$

$D_c$  is a parabola where the value of  $D_C$  at  $E_c$  is a minimum, i.e as  $E$  increases so does  $D_c$  and  $D_v$  is an inverted parabola like  $-x^2$  where the value of of  $D_v$  at  $E_v$  is also minimum, i.e as  $E$  increases  $D_v$  decreases.

### 4 Thermal Equilibrium and Fermi function

Idea: Given that we know that most of the electrons in the conduction band exist at the lowest energy level  $E_c$  and most of the holes in the valence band exist at the energy  $E_v$  here the idea is to examine the distribution of electrons and holes

**Analogy for Thermal Equilibrium** **Equilibrium** is defined as the lowest energy configuration in the *presence of thermal agitations*

In the presence of thermal agitations the particles can also take higher energy positions. However the system will be at its lowest possible energy consistent with the presence of “thermal” agitations. This is called **thermal equilibrium**

Electrons and holes receive and exchange energy from or with the crystal (phonons) and one another and every energy state in the conduction and valence band has a certain probability of being occupied by an electron.

### Quantification of occupancy of electrons and holes

To derive the expression for the probability of finding an electron at a certain energy: a statistical thermodynamic analysis with the assumption that the number of particles and the total system energy are held constant without regard to the specifics of how particles bounce off the atoms or one another yields the fermi-dirac distribution:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{kT}\right)}$$

Here,  $E_F$  is called the fermi energy and  $f(E)$  is the probability of a state at energy  $E$  being occupied by an electron. When  $E - E_F \gg kT$  the probability of a state being occupied decreases exponentially and the fermi-dirac distribution can be approximated using the boltzman distribution:

$$f(E) = e^{-\left(\frac{E-E_F}{kT}\right)}$$

At low energies  $E - E_F \ll kT$  the occupation probability approaches 1, i.e. low energy states tend to be fully occupied, or

$$f(E) \sim 1 - e^{-\left(\frac{E_F-E}{kT}\right)}$$

In this energy region, the probability of a state not being occupied, i.e. being occupied by a hole i.e.:

$$1 - f(E) \sim e^{-\left(\frac{E_F-E}{kT}\right)}$$

Therefore from the fermi-dirac distribution one the following important points occur:

1. The probability of occupancy at  $E_F$  is a 1/2
2. Probability approaches unity if  $E$  is much lower than  $E_F$  and approaches 0 at  $E$  much higher than  $E_F$
3. There exist only one  $E_F$  or one fermi level in a system at thermal equilibrium

## 5 Electron and Hole Concentrations

Consider an n-doped semiconductor with  $10^{16} \text{ cm}^{-3}$ : Question: what is the hole concentration? What is the carrier concentration in undoped semiconductor? To answer these questions one needs to derive a relationship between the Fermi-level and the carrier concentration

**Derivation of n and p from  $D(E)$  and  $f(E)$**  The electron and hole concentration is derive using the density of states and the fermi-dirac distribution:

$$n = \int_{E_C}^{\infty} f(E) D_c(E) dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_C}^{\infty} \sqrt{E - E_c} e^{\frac{-(E - E_F)}{kT}} dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{\frac{-(E_c - E_F)}{kT}} \int_{E_C}^{\infty} \sqrt{E - E_c} e^{\frac{-(E - E_c)}{kT}} d(E - E_c)$$

$$n = N_c e^{\frac{-(E_c - E_F)}{kT}}$$

$$N_c = 2 \left\{ \frac{2\pi m_n kT}{h^2} \right\}^{\frac{3}{2}}$$

$N_c$  is called the effective density of states of the conduction band. This is an important equation and can be remembered as follows:  $N_c$  is called the effective density of states. It is as if the energy states in the conduction band were effectively squeezed into a single energy level  $E_c$  which can hold  $N_c$  electrons (per cubic centimeter). Therefore the above equation is simply the product of  $N_c$  and the probability that an energy  $E_c$  is occupied.

The expression for the hole concentration is:

$$p = \int_{\text{Valence band bottom}}^{E_v} (1 - f(E)) D_v(E) dE$$

$$p = N_v e^{\frac{-(E_F - E_v)}{kT}}$$

$$N_v = 2 \left\{ \frac{2\pi m_p kT}{h^2} \right\}^{\frac{3}{2}}$$

$N_v$  is called the effective density of states of the valence band. The values of  $N_c$  and  $N_v$  are roughly the same ( $\sim 10^{19} \text{cm}^{-3}$ ) and differ only because of the difference in the effective mass. Table below contains the typical values for Silicon and GaAs

	Si	GaAs
$N_c (\text{cm}^{-3})$	2.8e19	4.7e17
$N_v (\text{cm}^{-3})$	1.04e19	7.0e18

**Fermi level and Carrier concentration** As is seen from the above equations of concentration of electrons and holes the fermi level is intrinsically linked to the concentration of electrons and holes, i.e. as the concentration of electrons the fermi energy moves closer to the conduction band energy, i.e.  $E_F = E_c$ , similarly for increase in hole concentration, i.e.  $E_F = E_v$ . When difference between the fermi energy and the conduction or the valence band is  $< 20 \text{meV}$ , i.e.  $\sim kT$  the Boltzman approximation is not quantitatively valid. This usually occurs when the semiconductor is heavily doped ( $> 10^{19} \text{cm}^{-3}$ ) or degenerate.

### The np product and the intrinsic carrier concentration

$$np = n_i^2 = N_c N_v e^{\frac{-E_g}{kT}}$$

The product above says that the  $np$  is constant for a given semiconductor and  $T$  and independent of the dopant concentration.  $n_i$  is called the intrinsic carrier concentration and is dependent on  $E_g$  and  $T$ . In the absence of doping  $n = p$  or the number of electrons excited due to thermal excitation results in an equivalent number of holes produced, i.e. electrons and holes are created in pairs.  $n_i$  at room temperature for Si is  $10^{10} \text{cm}^{-3}$  and for GaAs is  $10^7 \text{cm}^{-3}$ . GaAs is lower because of its larger band gap.

The constant product can be explained as follows: the electron-hole recombination rate is proportional to  $np$  product. When  $np = n_i^2$  the recombination rate happens to be equal to the rate of thermal generation of electron-hole pairs.

## 6 General theory of n and p

Shallow donors and acceptor levels ( $E_d$  &  $E_a$ ) are represented in the energy band as energy states and their occupancy by electrons is governed by the Fermi function.  $E_d$  is usually a few  $kT$  above  $E_F$ , the donor level is nearly empty of electrons, or the donor atoms are ionized, i.e. have lost all the electrons. This case is true for acceptors as well.

There are four types of charged species in a semiconductor: electrons, holes, positive donor ions and negative acceptor ions. Their densities are represented by  $n, p, N_a, N_d$ . In general one can assume that a sample is free of a net charge, i.e. the **Charge Neutrality** condition requires that the densities of the negative particles and positive particles are equal:

$$n + N_a = p + N_d$$

If  $N_d - N_a \gg n_i$  N-type

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n}$$

If  $N_d \gg N_a$

$$n = N_d$$

$$p = \frac{n_i^2}{N_d}$$

If  $N_a - N_d \gg n_i$  p-type

$$p = N_a - N_d$$

$$n = \frac{n_i^2}{p}$$

If  $N_a \gg N_d$

$$p = N_a$$

$$n = \frac{n_i^2}{N_a}$$

## 7 Carrier concentration at extremely high and low temperature

At very high temperatures,  $n_i$  is large and it is possible to have  $n_i \gg |N_d - N_a|$ , therefore  $n = p = n_i$ , i.e. the semiconductor becomes intrinsic

At very low temperatures  $E_F$  moves away from the dopant energy levels and most of the donors or acceptors remain nonionized. This phenomena is called freeze-out.

### Part II

# Motion and Recombination of Electrons and Holes

### Summary



1. In the presence of an electric field, charge carriers gain a **drift velocity** and produce a **drift current density** proportional to the electric field:

$$v_p = \mu_p E$$

$$v_n = -\mu_n E$$

$$J_{p,drift} = qp\mu_p E$$

$$J_{n,drift} = qn\mu_n E$$

$\mu_p$  and  $\mu_n$  are called the **hole** and **electron** mobility.

2. Both,  $\mu_p$  and  $\mu_n$  are determined by how frequently the carriers collide with phonons or dopant ions and lose their drift momentum. Mobilities are functions of temperature and total dopant concentration
3. Second important transport mechanism is **diffusion**. **Diffusion current density** is proportional to the gradient of the carrier concentration

$$J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

$$J_{n,diffusion} = qD_n \frac{dn}{dx}$$

$D_n$  and  $D_p$  are the electron and hole diffusion constants. Both drift and diffusion are perturbations to the same thermal motion, and both are slowed down by the same collisions that are responsible for the zigzag paths of the thermal motion.

4.  $D$  and  $\mu$  are related by the **Einstein** relations

$$D_n = \frac{kT\mu_n}{q}$$

$$D_p = \frac{kT\mu_p}{q}$$

5.  $J_{tot} = J_{drift} + J_{diffusion}$
6. Minority carrier concentration, e.g.  $p$  in an N-type semiconductor, can easily be increased from its **equilibrium concentration**  $p_0$  by orders of magnitude with light or doping, such that:

$$p = p_0 + p' \gg p_0$$

$p'$  is defined as the excess hole concentration

7. Charge neutral region,  $n' = p'$ . Charge non-neutrality will generate an electric field that causes the majority carriers to redistribute until neutrality is achieved

8. Electron and hole recombination rate is proportional to  $n' (=p')$ :

$$\text{Recombination rate} = \frac{n'}{\tau} = \frac{p'}{\tau}$$

$\tau$  is the recombination lifetime and ranges from nanoseconds to milliseconds for Si, depending on the density of trace metal impurities that form deep traps

9. **Quasi-Equilibrium** is condition when the excess minority carriers are present, the  $pn$  product can be orders of magnitude larger than  $n_i^2$ . This condition creates a non-equilibrium between electron and holes as two groups of particles. Within each group however the carriers are still in equilibrium among themselves and share one common (quasi) Fermi level at different locations. The following relationships elucidate this:

$$n = N_c e^{-\frac{(E_c - E_{Fn})}{kT}}$$

$$p = N_v e^{-\frac{(E_{Fp} - E_v)}{kT}}$$

$E_{Fn}$  and  $E_{Fp}$  are the quasi-Fermi levels of electrons and holes

## 8 Thermal Motion

The idea here being thermal effects alone can create an average kinetic energy of charge carriers. The motion resulting from this kinetic energy or velocity is random resulting in non steady electric current, but it does introduce thermal noise. The kinetic energy of electrons,  $E - E_c$  can be calculated as follows:

$$\text{Average electron kinetic energy} = \frac{\text{total kinetic energy}}{\text{number of electrons}} = \frac{\int f(E) D(E) (E - E_c) dE}{\int f(E) D(E) dE}$$

$$\text{Average electron kinetic energy} = \frac{3}{2} kT$$

The thermal velocity,  $v_{th}$ , of electrons and holes is:

$$v_{th} = \sqrt{\frac{3kT}{m}}$$

where  $m = m_n$  or  $m_p$ .

Electrons and holes move at thermal velocity but not in simple straight lines. Their direction of motion change frequently due to **collisions** or **scattering** with the imperfections in the crystal. The mean free time between collisions is typically 0.1 ps, and the distance between collisions is a few tens of nanometers. The net thermal velocity is zero.

## 9 Drift

Drift is the motion of charge carriers caused by an electric field. Drift is at play when voltages are applied to a semiconductor

### 9.1 Electron and hole mobilities

With the application of voltage the average velocity of carriers is no longer zero, in fact they carry a drift velocity. This drift velocity is superimposed on the thermal motion creating a net current flow. A large drift velocity is desired in semiconductors because it results in faster processing speeds. Due to the superimposing of the drift velocity on the thermal motion one can derive the drift velocity using the following assumptions: (a) mean free time between collisions is  $\tau_{mp}$  and that the carrier loses its entire drift momentum,  $m_p v$ , after each collision. Between collisions the drift momentum gained is equal to the force times the mean free time:

$$m_p \vec{v} = q \vec{E} \tau_{mp}$$

$$\vec{v} = \frac{q \vec{E} \tau_{mp}}{m_p} = \mu_p \vec{E}$$

$$\mu_p = \frac{q \tau_{mp}}{m_p}$$

Here  $\mu_p$  is the hole mobility or a metric of how mobile the holes are. This term depends on the mean free time between collision and inversely on the hole mass. For electrons one gets a similar expression:

$$\vec{v} = \frac{-q \vec{E} \tau_{mn}}{m_n} = -\mu_n \vec{E}$$

$$\mu_n = \frac{q \tau_{mn}}{m_n}$$

Typical values are given in the table below:

$\frac{cm^2}{Vs}$	Si	GaAs
$\mu_n$	1400	8500
$\mu_p$	470	400

GaAs has a high electron mobility because the electron effective mass is much smaller than Silicon's electron effective mass. Remember the effective mass is related to the curvature of the band.

## 9.2 Mechanism of Carrier Scattering

The mean free time between collision,  $\tau_{mp}, \tau_{mn}$  can vary significantly with temperature and doping concentration.

There are two causes of scattering in a crystal: phonon scattering and ionized impurity scattering.

**Phonon Scattering** - the physical mechanism of phonon scattering occurs due to electron and phonon interaction, i.e. the electron scatters due to interacting with the phonon or the crystal vibration. This crystal vibration distorts the periodic crystal structure thus scattering the electron wave. Therefore this brings in the concept of mobility of phonons due to scattering which is:  $\mu_{phonon} = \frac{q\tau_{ph}}{m}$  where  $\tau_{ph}$  is the mean free time of phonon scattering. The mean free time of phonon scattering is inversely proportional to phonon density and electron speed (or thermal velocity). Phonon density is proportional to temperature. The mobility of phonons therefore is:

$$\mu_{phonon} \propto \tau_{ph} \propto \frac{1}{phonon\ density \times carrier\ thermal\ velocity} \propto \frac{1}{T\sqrt{T}} \propto T^{-\frac{3}{2}}$$

The above equation shows that the scattering mobility decreases as the temperature increases or as the temperature increases the scattering life time goes from being very long to very short, which is as expected.

**Impurity Scattering** - Physically scattering rises because the dopant ions are fixed charge in the semiconductor crystal, therefore due to coulombic forces they can make the electrons and holes change the direction of motion. Electrons or holes can be scattered by either a donor (positive) or an acceptor (negative) ion. The mobility due to this type of scattering is inversely proportional to the donor or acceptor ion concentrations and proportional to  $T^{\frac{3}{2}}$ . The mobility increases at higher temperature because the higher thermal velocity means the charge carriers fly by the ions in a shorter time therefore resulting in lower perturbation to the electrons motion.

$$\mu_{impurity} \propto \frac{T^{\frac{3}{2}}}{N_a + N_d}$$

In the event of more than one scattering mechanism the total scattering rate and therefore the total mobility are determined by the sum of their inverses:

$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

The mobilities may be expressed as:

$$\mu_p = \frac{420}{1 + \left[ \frac{(N_a + N_d)}{1.6e17} \right]^{0.7}} + 50$$

$$\mu_n = \frac{1318}{1 + \left[ \frac{(N_a + N_d)}{1.0e17} \right]^{0.85}} + 92$$

From the above equations the inverse proportionality to the dopant density is not followed at the limit of very  $N_a$  or  $N_d$ . The reason is free carrier screening. When the carrier concentration is large the carriers can distribute themselves to partially screen out the coulombic field of dopant ions.

**Velocity Saturation and Ballistic Transport** In small device one can get large applied electric field however the drift velocity will always saturate at around  $10^7 \frac{cm}{s}$ . This is because of optical phonons. When the carrier kinetic energy exceeds the optical phonon energy,  $E_{opt}$ , it generates an optical phonon and loses its kinetic energy.

Mobility and even velocity saturation are concepts that describe the carrier motion averaged over many scattering events. These concepts become fuzzy when dealing with devices whose sizes are smaller than the mean free path. The motion of carriers in a nearly scattering-free environment is called **ballistic transport**.

$$\frac{mv_{sat}^2}{2} = E_{opt}$$

$$v_{sat} = \sqrt{\frac{2E_{opt}}{m}}$$

$E_{opt}$  is the optical phonon energy and is usually 40 meV, which puts  $v_{sat} \sim 10^7 \frac{cm}{s}$ . **Velocity saturation** has deleterious effects on device speeds

### 9.2.1 Drift Current and Conductivity

$$\vec{J}_{p,drift} = qp\vec{v}$$

$$\vec{J}_{p,drift} = qp\mu_p\vec{E}$$

$$\vec{J}_{n,drift} = qp\mu_n\vec{E}$$

$$\vec{J}_{drift} = \vec{J}_{n,drift} + \vec{J}_{p,drift} = (qp\mu_n + qp\mu_p)\vec{E} = \sigma\vec{E}$$

where  $\sigma$  is the conductivity of the semiconductor

## 10 Diffusion Current

Diffusion current is the second mechanism of current flow. This type of current flow is not considered in metals because of their high conductivities. In semiconductors it plays a role because of the low conductivity and the ease of creating a nonuniform carrier densities.

Diffusion is the result of particles undergoing thermal motion, i.e. particle movement from a point of higher particle density towards a lower particle density. Mathematically we say the rate of movement by diffusion is proportional to the concentration gradient, i.e.

$$\vec{J}_{n,diffusion} \propto \frac{dn}{dx}$$

$$\vec{J}_{n,diffusion} = qD_n \frac{dn}{dx}$$

$$\vec{J}_{p,diffusion} = -qD_p \frac{dp}{dx}$$

The negative sign is due to sign convention of taking the flow of current due to the diffusion of electrons as positive. Therefore the holes will diffuse in the opposite direction and the current produced will also be in the opposite direction.

$$\vec{J}_n = \vec{J}_{n,drift} + J_{n,diffusion} = qn\mu_n\vec{E} + qD_n \frac{dn}{dx}$$

$$\vec{J}_p = \vec{J}_{p,drift} + J_{p,diffusion} = qp\mu_p\vec{E} - qD_p \frac{dp}{dx}$$

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

## 11 Relation between Energy band diagram and $V, \vec{E}$

A voltage applied across a piece of semiconductor alters the band diagram. **By definition, a positive voltage raises the potential energy of all positive charges and lowers the potential energy of a negative charge.** This means the band diagram is higher where the voltage is lower or the band diagram is lower where the voltage is higher. *Point to remember is that  $E_c$  and  $E_v$  vary in the opposite direction from the voltage.  $E_c$  and  $E_v$  are higher where the voltage is lower.* That is to say

$$E_c(x) = constant - qV(x)$$

One can think of the following analogy: *electrons roll downhill like stones in the energy band diagram and the holes float up like bubbles.*

## 12 Einstein relationship between $D$ and $\mu$

Considering a semiconductor at equilibrium, meaning the total current density is zero, i.e.  $\vec{J}_n = \vec{J}_p = 0$ . From the above equations:

$$0 = qn\mu_n\vec{E} + qD_n\frac{dn}{dx}; n = N_c e^{-\frac{(E_c - E_F)}{kT}}$$

$$0 = qn\mu_n\vec{E} - qn\frac{qD_n}{kT}\vec{E}$$

$$D_n = \frac{kT}{q}\mu_n$$

At equilibrium the drift and diffusion currents perfectly cancel each other out for an arbitrary doping profile. A close relationship between  $D$  and  $\mu$  becomes possible when one realizes that all scattering mechanisms (phonon and impurity scattering) that impede electron drift would also impede electron or hole diffusion. The equation for hole is:

$$D_p = \frac{kT}{q}\mu_p$$

## 13 Electron-Hole recombination

Electron and hole concentration introduced are the equilibrium carrier concentration and are denoted as:  $n_o$  &  $p_o$ . These concentration can be different if light shines on the sample and generate electron and holes. The differences are known as **excess carrier concentration** denoted by  $n'$  &  $p'$ . If  $n'$  &  $p'$  are created by light then they are equivalent because electrons and holes are created in pairs. If  $n'$  &  $p'$  are introduced by other means then they will still be equal because of charge neutrality. A charge neutral sample can be written as:  $n' = p'$ . The concentration of electrons and holes as a result of the excess carrier concentration can be written in mathematical form as:

$$n = n_o + n'$$

$$p = p_o + p'$$

If the light is suddenly turned off  $n'$  &  $p'$  will decay with time until they become zero and  $n$  &  $p$  return to their equilibrium values  $n_o$  &  $p_o$ . The process of decay is **recombination** where by an electron and a hole recombine and annihilate each other. The time constant of decay is called the **recombination time** or **carrier lifetime**  $\tau$

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = \frac{p'}{\tau}$$

The recombination rate (per cubic centimeter per second) is proportional to  $n'$  &  $p'$ .

$$\text{Recombination rate} = \frac{n'}{\tau} = \frac{p'}{\tau}$$

$\tau$  has dimensions of time and is typically around  $1\mu s$  in Silicon. It may range from 1ns to 1ms, depending on the density of trace metal impurities such as Au and Pt, which form traps in the band gap with several energy levels deep in the band gap. These **deep traps** can capture electrons and holes to facilitate recombination and thus shorten the recombination time. Too small  $\tau$  is bad for device leakage current thus extreme cleanliness is maintained in the semiconductor fabrication plants partly to avoid these metallic contaminants.

In direct band gap semiconductors one gets radiation being emitted when electrons and holes recombine.

## 14 Thermal generation

The reverse process of recombination is **thermal generation**. At any nonzero temperature, electron-hole pairs are constantly being generated and lost by recombination.